Name: Claire Li   Date: Nov. 20

$t_n = a + (n-1)d$ **Section 2.1 Arithmetic Sequences and Series**

1. Given each arithmetic sequence, find the value of the missing term:

a) $4, 10, 16, 22, \ldots t_{10}$ $t_n = 4 + 9 \times 6$ $= \boxed{58}$	b) $-24, -12, 0, 12, \ldots t_9$ $t_n = -24 + 8 \times 12$ $= \boxed{72}$
c) $\frac{24}{5}, \frac{14}{5}, \frac{4}{5}, \frac{-6}{5}, \ldots t_8$ $t_n = \frac{24}{5} + 7 \times (-2)$ $= \frac{24}{5} - \frac{70}{5} = \boxed{-\frac{46}{5}}$	d) $\frac{22}{3}, \frac{47}{6}, \frac{25}{3}, \frac{53}{6}, \ldots t_8$ $t_n = \frac{22}{3} + 7 \cdot \frac{3}{6}$ $\boxed{\frac{65}{6}}$
e) $a = -4,\ d = 5, \ldots t_6$ $t_n = -4 + 5 \cdot 5$ $= \boxed{21}$	f) $2x+1, 4x, 5x+2, \ldots t_6$ $7 \quad 12 \quad 17$ $4x - 2x - 1 = 2x - 1$ $5x + 2 - 4x = x + 2$ $2x - 1 = x + 2$ $x = 3$ $t_n = 7 + 5 \times 5$ $= 32$

2. Given each arithmetic sequence, find out how many terms there are and the sum of the sequence. Show all your work and steps for each question:

a) $5, 9, 13, \ldots 209$ $209 = 5 + (n-1)4$ $204 = 4n - 4$ $208 = 4n$ $n = 52$ $S = \frac{(209+5)52}{2}$ $= 5564$ Number of terms: 52 Sum: 5564	b) $-210, -207.5, -205, \ldots 45$ $45 = -210 + (n-1)2.5$ $255 = 2.5n - 2.5$ $257.5 = 2.5n$ $n = 103$ $\frac{(45-210)103}{2} =$ Number of terms: 103 Sum: -8497.5
c) $\frac{1}{2}, \frac{7}{6}, \frac{11}{6}, \ldots \frac{29}{2}$ $\frac{29}{2} = \frac{1}{2} + (n-1)\frac{4}{6}$ $87 = 3 + 4n - 4$ $88 = 4n$ $n = 22$ $\left(\frac{3}{6} + \frac{29}{2}\right)\frac{22}{2} =$ $= \frac{3+87}{6} \cdot 11 = 165$ Number of terms: 22 Sum: 165	d) $\frac{4}{5}, \frac{2}{15}, \frac{-8}{15}, \ldots \frac{-126}{5}$ $-\frac{126}{5} = \frac{4}{5} + (n-1)\left(-\frac{2}{3}\right)$ $-378 = 12 - 10(n-1)$ $-378 = 22 - 10n$ $-400 = -10n \quad n = 40$ $\left(\frac{4}{5} + -\frac{126}{5}\right) \times \frac{40}{2}$ $= -\frac{122}{5} \times 20$ $= -488$ Number of terms: 40 Sum: -488
e) $x-8, x-2, 2x-1, \ldots 35x+2$ $x + 4 = 2x - 1$ $5 = x$ $-3 \quad 3 \quad 8 \cdots 177$ $177 = -3 + (n-1)6$ $177 = -3 + 6n - 6$ $186 = 6n$ $n = 31$ $(-3+177)\frac{31}{2}$ $= 2697$ Number of terms: 31 Sum: 2697	f) $7 - x^2, x + 7, 2x^2 - 1, \ldots 6x^3 + 47$ $x + 7 - 7 + x^2 = 2x^2 - 1 - x - 7$ $0 = x^2 - 2x - 8$ $0 = (x-4)(x+2)$ $x = 4$ or -2 ① $3 \quad 5 \quad 7 \quad 95$ $95 = 3 + (n-1)2$ $95 = 3 + 2n - 2$ $94 = 2n$ $n = 47$ $\frac{(3+95)47}{2}$ $= 2303$ ② $-9 \quad 11 \quad 31 \quad 431$ $431 = -9 + (n-1)20$ $440 = 20(n-1)$ $22 = n - 1$ $23 = n$ $\frac{(431-9)23}{2}$ $= \frac{422 \cdot 23}{2}$ $= 211 \cdot 23$ $= 4853$ Number of terms: ① 47 ② 23 Sum: 2303, 4853

3. What is the arithmetic mean of 3, 5, 9, 12, and 21?

a) 3 b) 5 c) 9 d) 10 e) 18

4. What is the seventh term of an arithmetic sequence with a first term of nine and a common difference of twelve?

$9+(7-1)12$
$=9+72$
$=\boxed{81}$

5. If the first four terms of an arithmetic sequence are a, $2a$, b, and $a-6-b$ for some numbers a and b, then the value of the 100th term is

a) -100 b) -300 c) 150 d) -150 e) 100

-1 -2 -3 -4

$2a+a=b$ $\quad b+a=a-6-b$
$3a=b$ $\quad 6=-2b$
$a=\frac{b}{3}$ $\quad b=-3$

6. Let T_n be the nth term of a sequence, where "n" is a natural number. Which of the following is/are arithmetic sequences?

i) $T_n = 2n-3$ ii) $T_n = 3^n$ X iii) $T_n = n^2$ X

$T_n = a+(n-1)d$

7. In an arithmetic sequence, the 3rd term is "x" and the 5th term is "y". Which expression is the first term?

i) $2x-y$ ii) $2x+y$ iii) $2y-x$ iv) $2y+x$

$d=\frac{y-x}{2}$ $\quad x-(y-x)$ $\quad 2x-y$
$x-y+x$

8. Consider the following arithmetic sequence, which term is the first positive term? $-16, -14.75, -13.5\ldots$

$+1.25$

$-16+(n-1)1.25>0$ $\quad n>13.8$ $\quad n=\boxed{14}$
$-16+1.25n-1.25>0$
$1.25n>17.25$

9. the nth term of an arithmetic sequence is given by the formula $T_n = 3n-1$. What is the sum of the first "n" terms of this series?

$a+(n-1)d=3n-1$

i) $\frac{n(2n+1)}{2}$ ii) $\frac{n(2n-1)}{2}$ iii) $\frac{n(3n+1)}{2}$ iv) $\frac{n(3n-1)}{2}$

$a+dn-d=3n-1$ $\quad d=3$ $\quad a+3n-3=3n-1$ $\quad a=2$

$\frac{(2+3n-1)n}{2}=\frac{(3n+1)n}{2}$

10. An increasing sequence is formed so that the difference between consecutive terms is a constant. If the first four terms of this sequence are x, y, $3x+y$, and $x+2y+2$, then the value of $y-x$ is

a) 2 b) 3 c) 4 d) 5 e) 6

$y-x=3x+y-y$ $\quad x+2y+2-3x-y=y-x$ $\quad y=4x=8$
$y=3x+x$ $\quad 2=-x+3x-x$ $\quad y-x=\boxed{6}$
$y=4x$ $\quad x=2$

11. Four numbers are in an arithmetic sequence and their sum is 82. The 4th term is greater than the 2nd term by how much?

$\frac{(a+(a+3d))4}{2}=82$ $\quad 2d=\boxed{14}$

$2a+3d=41 \rightarrow \begin{cases} a=10 \\ d=7 \end{cases}$

12. In a sequence of positive number, each term after the first two terms is the sum of *all the previous terms*. If the first term is a, the second term is 2, and the sixth term is 56, then the value of a is

a) 1 b) 2 c) 3 d) 4 e) 5

$a \quad 2 \quad a+2 \quad 2a+4 \quad 4a+8 \quad 8a+16$

$8a+16=56$

$8a=40$

$a=5$

13. The sum of the arithmetic series (-300)+(-297)+(-294)+...+306+309 is

a) 309 b) 927 c) 615 d) 918 e) 18

$309=-300+(n-1)3$

$609=3n-3$

$612=3n$

$n=204$

$\frac{(-300+309)204}{2}$

$=9\times102$

$=918$

14. What is the sum of all the integers between 1 and 300 which are divisible by 7?

$7 \quad 14 \quad 28 \cdots 294$

$294=7+(n-1)7$

$294=7n$

$n=42$

$(7+294)\frac{42}{2}$

$=\boxed{6321}$

15. A nine term arithmetic sequence $a_1, a_2, \ldots, a_8, a_9$ satisfies $a_5+a_7=-17$ and $a_4+a_6=1$. What is the sum of the terms of the sequence?

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \quad a_8 \quad a_9$

$a_5+a_7=2a_5+2d=-17$

$2a_5-18=-17$

$2a_5=1$

$a_5=\frac{1}{2}$

$a_3+a_7=a_4+a_6=1$

$a_5+a_7=-17$

$a_5-a_3=-18$

$d=-9$

$S=4(a_4+a_6)+a_5$

$=4+\frac{1}{2}$

$\boxed{\frac{9}{2}}$

16. The arithmetic sequence a, $a+d$, $a+2d$, $a+3d$, ..., $a+(n-1)d$ has the following properties:

- When the first, third, and fifth, and so on terms are added, up to and including the last term, the sum is 320.
- When the first, fourth, seventh, and so on, terms are added, up to and including the last term, the sum is 224.

$a \quad a+3d \quad a+6d$

What is the sum of the whole sequence?

a) 656 b) 640 c) 608 d) 704 e) 672

$\frac{1}{2}\frac{n+1}{2}(a+a+(n-1)d)=320$

$(n+1)(2a+nd-d)=1280$

$2a+nd-d=X$

$\frac{1}{2}\frac{n+2}{2}(a+a+(n-1)d)=224$

$(n+1)(2a+nd-d)=1344$

$(n+1)X=1280$

$(n+2)X=1344$

$\frac{n+2}{n+1}=\frac{672}{640}$

$\frac{n+2}{n+1}=\frac{21}{20}$

$21n+21=20n+40$

$n=19$

$a+a+(n-1)d=\frac{1280}{20}$

$=64$

$\frac{64\times19}{2}=608$

17. A sequence of three real number forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting number form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

a) 1 b) 4 c) 36 d) 49 e) 81

$9 \quad 9+d \quad 9+2d$

$9 \quad 11+d \quad 29+2d$

$\frac{29+2d}{11+d}=\frac{11+d}{9}$

$261+18d=121+11d+11d+d^2$

$0=d^2+4d-140$

$d=(d-10)(d+14)$

$d=10$ or -14

$9 \quad -3 \quad 1$

$-\frac{1}{3} \quad -\frac{1}{3}$

$-6-6-6$ $\quad -6-6-6$

5. (a) In the series of odd numbers $1+3+5-7-9-11+13+15+17-19-21-23$ ... the signs alternate every three terms, as shown. What is the sum of the first 300 terms of the series?

$(300\div 6)\times -18 = \boxed{-900}$

An arithmetic sequence $a,\ a+d,\ a+2d,\ \ldots$ is a sequence in which successive terms have a common difference d. For example, $2, 5, 8, \ldots$ is an arithmetic sequence with common difference $d = 3$ because $5-2 = 8-5 = 3$.

(a) If $x-1$, $2x+2$ and $7x+1$ are the first three terms of an arithmetic sequence, determine the value of x.

$2x+2-x+1 = 7x+1-2x-2$ $\quad 4=4x$

$x+3 \quad = 5x-1$ $\quad x=\boxed{1}$

(b) For the value of x from (a), what is the middle term of the arithmetic sequence $x-1,\ 2x+2,\ 7x+1,\ \ldots,\ 72$?

$0 \quad 4 \quad 8 \quad 72$

$72=0+(n-1)4$ $\quad 0+9\times 4=\boxed{36}$

$76=4n$

$n=19$

$\frac{72+0}{2}=36$

3. (a) In an arithmetic sequence with five terms, the sum of the first two terms is 2 and the sum of the last two terms is -18. What is the third term of this sequence? (An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3, 5, 7, 9, 11 is an arithmetic sequence with five terms.)

$2 \quad -16 \quad 16$

$\frac{(2+-18)}{4} = -4 = \boxed{-4}$

$b-a=c-b$

$2b=a+c$

(b) Suppose that a, b, c are three consecutive terms in an arithmetic sequence. Prove that a^2-bc, b^2-ac, and c^2-ab are also three consecutive terms in an arithmetic sequence.

$b^2-ac-a^2+bc = c^2-ab-b^2+ac$

$0 = a^2-2b^2+c^2+2ac-bc-ab$

(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3, 5, 7 is an arithmetic sequence with three terms.)

$0 = (a+c)^2-2b^2-bc-ab$

$= (2b)^2-2b^2-(a+c)b$ $\quad 0=0 \quad \therefore QED.$

$= 4b^2-2b^2-2b^2 = 0$

4. (a) In an arithmetic sequence, the first term is 1 and the last term is 19. The sum of all the terms in the sequence is 70. How many terms does the sequence have?

(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3, 5, 7, 9 is an arithmetic sequence with four terms.)

$\frac{(1+19)n}{2}=70$

$10n=70$ $\quad$ 7 terms.

$n=\boxed{7}$

Name: Claire Li   Date: Nov. 21

Section 2.2 Geometric Sequences

1. Which of the following sequences is geometric. Indicate YES or NO: If YES, indicate the common ratio:

a) 2, 4, 6, 8, 10..... No	b) 0.25, 0.50, 1.0, 02.0, 04.0 Yes. R=2
c) $\frac{2}{3}, \frac{-1}{3}, \frac{1}{6}, \frac{-1}{12}, \frac{1}{24}$ Yes. $R=-\frac{1}{2}$ $-\frac{1}{2}$	d) $\frac{27}{32}, \frac{9}{16}, \frac{3}{8}, \frac{1}{4}, \frac{1}{6}$ Yes, $R=\frac{2}{3}$ $\frac{2}{3}$
e) 0.75, −0.75, 0.75, −0.75, 0.75 Yes, R=−1 −1	f) $a+b,\ a+b^2,\ a+b^3,\ a+b^4,\ a+b^5$ No
g) $\frac{a}{b}, -\frac{a^2}{b^3}, \frac{a^3}{b^6}, -\frac{a^4}{b^9}, \frac{a^5}{b^{12}}$ No. $-\frac{a}{b^2}$	h) $\frac{c^2}{d}, \frac{d}{c^2}, \frac{c^2}{d}, \frac{d}{c^2}$ No $\frac{d}{c^2}, \frac{d}{c^2} = \frac{d^2}{c^4}$

2. If the following is a geometric sequence, indicate the number of terms:

a) 6, 12, 24, ..., 3072 $3072=6\times2^{n-1}$ $512=2^{n-1}$ $2^{n-1}=2^9$ $n=\boxed{10}$	b) 24, 12, 6, ..., $\frac{3}{512}$ $\frac{3}{512}=24\times\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{2^9}=2^3\times\left(\frac{1}{2}\right)^{n-1}$ $\frac{1}{2^{12}}$ $n-1=12$ $n=\boxed{13}$
c) $\sqrt{3},\ -3,\ 3\sqrt{3},$ ...., $243\sqrt{3}$, $R=-\sqrt{3}$ $243\sqrt{3}=\sqrt{3}\times(-\sqrt{3})^{n-1}$ $243=(-\sqrt{3})^{n-1}$ $\sqrt{3}^{10}=(\sqrt{3})^{n-1}$ $n-1=10$ $n=\boxed{11}$	d) $\frac{1}{8}$, −0.25, 0.5, ..., −1024 $R=-2$ $-1024=\frac{1}{8}\times(-2)^{n-1}$ $-2^{13}=(-2)^{n-1}$ $n-1=13$ $n=\boxed{14}$
e) 396, −132, 44, ..., $\frac{44}{729}$ $R=-\frac{1}{3}$ $\frac{44}{729}=396\times\left(-\frac{1}{3}\right)^{n-1}$ $\frac{1}{3^8}=\left(-\frac{1}{3}\right)^{n-1}$ $n-1=8$ $n=\boxed{9}$	f) $\frac{a^3}{b},\ a^2,\ ab,$ ......, $\frac{b^{15}}{a^{13}}$ $R=\frac{b}{a}$ $\frac{a^3}{b}\cdot\left(\frac{b}{a}\right)^{n-1}=\frac{b^{15}}{a^{13}}$ $\left(\frac{b}{a}\right)^{n-1}=\frac{b^{16}}{a^{16}}$ $n-1=16$ $n=\boxed{17}$
g) 2048, 512, 128, ..., $\frac{1}{2048}$ $R=\frac{1}{4}$ $\frac{1}{2048}=2048\cdot\left(\frac{1}{4}\right)^{n-1}$ $\frac{1}{4^{11}}=\frac{1}{4}^{n-1}$ $n-1=11$ $n=\boxed{12}$	h) $x-3,\ x,\ 3x+4,$ ...., $(x+4)^6$ $\frac{x}{x-3}=\frac{3x+4}{x}$ $x=4$ or $-\frac{3}{2}$ $x^2=3x^2+4x-9x-12$ $0=2x^2-5x-12$ $0=(2x+3)(x-4)$ 1) $x=4$ $4^6=1\times4^{n-1}$ $n=\boxed{7}$ 2) $x=-\frac{3}{2}$ $\left(\frac{5}{2}\right)^6=-\frac{9}{2}\times\left(\frac{1}{3}\right)^{n-1}$ ✗

3. Given the information of a geometric sequence, find the indicated unknown value:

a) $a=-3,\ r=5,\ t_4=$ −1875 $-3\times5^4=-1875$	b) $S_2=5,\ S_4=85,\ r=$ $\pm\sqrt{13}$ $a\quad ar\quad ar^2\quad ar^3$ $a(r+1)=5$ $a(r^3+r^2+r+1)=85$ $\frac{r^3+r^2+r+1}{r+1}=13$ $\frac{r^2(r+1)+r+1}{(r+1)}=13$ $r^2+1=13$ $r=\pm\sqrt{13}$
c) $S_3=26,\ S_\infty=27,\ r=$ $\sqrt[3]{\frac{1}{26}}$ $a(1+r+r^2)=26$ $a(1+r+r^2+r^3+r^4+r^5)=27$ $\frac{r^5+r^4+r^3+r^2+r+1}{r^2+r+1}=\frac{27}{26}$ $r^3+1=\frac{27}{26}$ $r^3=\frac{1}{26}$ $r=\sqrt[3]{\frac{1}{26}}$	d) $t_4=\frac{4}{27},\ t_7=\frac{32}{729},\ S_6=$ $\boxed{\frac{665}{486}}$ $ar^3=\frac{4}{27}\quad ar^6=\frac{32}{729}$ $r^3=\frac{32}{729}\div\frac{4}{27}=\frac{8}{27}$ $r=\frac{2}{3}$ $\frac{1}{2}\quad\frac{1}{3}\quad\frac{2}{9}\quad\frac{4}{27}$

$\frac{1}{2}\left(1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+\frac{32}{243}\right)$

$=\frac{1}{2}\cdot\frac{243+162+108+72+48+32}{243}=\boxed{\frac{665}{486}}$

$S_n = \frac{a(r^n-1)}{(r-1)}$

e) $a=12,\ r=2,\ S_n=762,\ n=?$ $S_n=\frac{a(r^n-1)}{r-1}$ $762=12(2^n-1)$ $\frac{127}{2}+1=2^n$ $\frac{129}{2}=2^n$	f) $t_2=3x,\ t_3=2x-1,\ t_4=7x+8,\ S_6=$ $4x^2-4x+1=21x^2+24x$ $0=17x^2+28x-1$ $x=\frac{-28\pm2\sqrt{213}}{4}$ $=-14\pm\sqrt{218}$
g) $a=x+2,\ t_2=3x,\ t_3=x^2+8,\ S_5=$ 6 12 24 $\frac{3x}{x+2}=\frac{x^2+8}{3x}$ 1 -3 9 $9x^2=x^3+8x+2x^2+16$ $7x^2=x^3+8x+16$ $0=x^3-7x^2+8x+16$ $(x+1)(x-4)^2=0$ $x=4$ or -1 $x=4$ $S_5=\frac{6(2^5-1)}{1}=\boxed{186}$ $x=-1$ $S_5=\frac{1(-3^5-1)}{-2}=\boxed{122}$	h) $t_3=12,\ t_4=k,\ t_5=48,\ k=$ 24 $r=\sqrt{\frac{48}{12}}=2$

4. Bacteria grows by division every 20 minutes. One bacterium splits to two in 20 minutes and then it becomes four another 20 minutes later. If one bacterium is put into a culture at 8:30am and the culture is covered at 7:00pm, when was the culture have covered? How many bacteria is there at 7pm?

5. Geometric Means between two terms:
   a. Determine two geometric means between 12 and 48.

   $\pm\sqrt{12\times48}$

   $=\pm12\sqrt{4}$

   $=\pm24$

   $\boxed{\pm24}$

   b. Determine two geometric means between x and y

   $\boxed{\pm\sqrt{xy}}$

6. What value of "x" in $x,\ 2x+2,\ 3x+3$ will form a geometric sequence?

   $\frac{2x+2}{x}=\frac{3x+3}{2x+2}$

   $4x^2+8x+4=3x^2+3x$

   $x^2+5x+4=0$

   $(x+4)(x+1)=0$

   $x=\boxed{-1 \text{ or } -4}$

7. If "a" , "b" and "c" are in a geometric sequence, which are following are also a geometric sequence?

   i) $\frac{1}{a},\ \frac{1}{b},\ \frac{1}{c}$ $b=ar$ $c=ar^2$

   $\frac{1}{a}\ \frac{1}{ar}\ \frac{1}{ar^2}$ ✓ $R=\frac{1}{r}$

   ii) $c,\ -b,\ a$

   $ar^2\ -ar\ a$ ✓ $R=-\frac{1}{r}$

   iii) $3^a,\ 3^b,\ 3^c$ ✓

   $3^a\ 3^{ar}\ 3^{ar^2}$

   $3^a\ 3^a\cdot3^r\ 3^a\cdot(3^{r^2})$ $R=3^r$

8. Determine the value of "x" which makes $3, 3^x, 3^{x-5}$ a geometric sequence?

   $3(3^{x-5})=(3^x)^2$

   $3^{x-4}=3^{2x}$

   $x-4=2x$

   $x=-4$

9. If T_n is the value of the nth term, with $T_n + T_{2n} = x$ and $T_{2n} + T_{3n} = y$, where "x" and "y" are both not equal to zero, which expression is equal to the common ratio?

i) $\left(\frac{x}{y}\right)^n$ ii) $\left(\frac{y}{x}\right)^n$ iii) $\left(\frac{x}{y}\right)^{\frac{1}{n}}$ iv) $\left(\frac{y}{x}\right)^{\frac{1}{n}}$

$T_n = ar^{n-1}$

$T_{2n} = ar^{2n-1}$

$T_{3n} = ar^{3n-1}$

$x = ar^{n-1}(1+r^n)$

$y = ar^{2n-1}(1+r^n)$

$\frac{y}{x} = \sqrt[n]{r^n}$

10. If the first two terms of a geometric sequence are $\sqrt{3}$, $\sqrt[3]{3}$ what is the 3rd term?

$r = \frac{\sqrt[3]{3}}{\sqrt{3}} = 3^{\frac{1}{3}} \div 3^{\frac{1}{2}}$

$= 3^{\frac{1}{3}-\frac{1}{2}}$

$= 3^{-\frac{1}{6}}$

$\sqrt[3]{3} \times 3^{-\frac{1}{6}}$

$= 3^{\frac{1}{3}-\frac{1}{6}}$

$= \boxed{3^{\frac{1}{6}} = \sqrt[6]{3}}$

11. If $t_1 = x+6$, $t_2 = 2-7x$, and $t_3 = -20x-4$ are three consecutive terms in a geometric sequence, determine the value(s) of "x"

$x = \frac{-96 \pm \sqrt{96^2}}{69^2}$

$(2-7x)^2 = (x+6)(-20x-4)$

$-49x^2 + 28x - 4 = 20x^2 + 120x + 4x + 24$

$-69x^2 - 96x - 28 = 0$

$69x^2 + 96x + 28 = 0$

12. If the nth term of a geometric sequence is given by the formula $S_n = 3\left(\frac{1}{5}\right)^{n-1}$. What is the common ratio?

$= ar^{n-1}$

$r = \boxed{\frac{1}{5}}$

13. In a geometric sequence, $t_6 = -160$ and $t_9 = 1280$, find the value of t_1.

$\begin{cases} 1280 = t_1 r^8 \\ -160 = t_1 r^5 \end{cases}$

$-8 = r^3$

$r = -2$

$-160 = -32 t_1$

$t_1 = \boxed{5}$

14. A ball is dropped from a height of 2.0m. After each bounce, it rises to 63% of its previous height. Write a general equation for the height after each bounce. What height does the ball rach after 5 bounces? $\times 0.63$

2 _ _ _ _ _ 6th

$2(0.63)^5 \approx \boxed{0.20}\,m.$

15. If $t_5 = 3x+2$ ~~and $t_7 = 7x-22$~~ with a common ratio of $r=-3$, determine t_6 *and* t_8.

$9(3x+2) = 7x-22$ $\qquad t_7 = -36$ $\qquad t_6 = 18$

$27x+18 = 7x-22$ $\qquad \boxed{18 \quad -36 \quad 72}$ $\qquad t_8 = 72$

$20x = -40$

$x = \boxed{-2}$

16. Determine t_2 of a geometric sequence if $t_4 + t_5 = -3$ and $t_3 + t_4 = -6$

$t_4 = ar^3$ $\qquad ar^3 + ar^4 = -3$ $\qquad \frac{ar^3}{ar^2} = \frac{-3}{-6}$ $\qquad a \cdot \frac{1}{8} \cdot \frac{3}{2} = -3$

$t_5 = ar^4$ $\qquad ar^3(1+r) = -3$ $\qquad a = -16$

$t_3 = ar^2$ $\qquad ar^2(1+r) = -6$ $\qquad r = \frac{1}{2}$ $\qquad -16 \cdot \frac{1}{2} = \boxed{-8}$

17. Given that $t_1 = 0$ and each term afterwards is equal to half of the previous term plus 1. Give an expression for all terms in the sequence in terms of "n"

$0 \quad 1 \quad \frac{3}{2} \quad \frac{7}{4}$

$a \cdot \frac{a+2}{2} \quad \frac{a+2+4}{4} \quad \frac{a+2+4+8}{8}$ $\qquad \frac{0+2^1+2^2+2^3+\cdots+2^{n-1}}{2^{n-1}}$ $\qquad \frac{2(2^{n-1}-1)}{2^{n-1}}$ $\qquad \boxed{\frac{2^{n-1}-1}{2^{n-2}}}$

$\frac{a+2^1+2^2+2^{n-1}4}{2^{n-1}}$

18. If a, b and c are positive, consecutive terms of a geometric sequence $\left(\textit{that is } \frac{c}{b} = \frac{b}{a}\right)$, then how would the graph of $y = ax^2 + bx + c$ look like? Choose your answer and explain why:

4 2 4

a) ~~A curve that intersects the~~ x-axis at two distinct points b) Entirely below the x-axis
c) Entirely above the x-axis d) A straight line e) Tangent to the x-axis

$b^2 = ac$ $\qquad \Delta < 0$

$\Delta = b^2 - 4ac$ $\qquad \therefore$ no roots

$= ac - 4ac$ $\qquad \therefore$ doesn't cross the x-axis.

$= -3ac$

19. A sequence of numbers has 6 as its first term, and every term after the first is defined as follows: If a term, t, is even, the next term in the sequence is $\frac{1}{2}t$. If a term, s, is odd, the next term is $3s+1$. Thus, the first four terms in the sequence are 6, 3, 10, 5. The 100^{th} term is

a) 1 b) 2 c) 3 d) 4 e) 6

$6 \quad 3 \quad 10 \quad 5 \quad 16 \quad 8 \; (4 \;\; 2 \;\; 1) \; 4 \;\; 2 \;\; 1 \quad 4 \;\; 2 \;\; 1 \; \cdots$

$\left\lfloor \frac{100-6}{3} \right\rfloor = \left\lfloor \frac{94}{3} \right\rfloor = 31$

20. For the previous question, for any value that you pick as the first term, what value will the last term in the sequence be? (Challenge: Proof)

always end in 421421. so the last term is $\boxed{1}$

21. Consider sequences of positive real numbers of the form x, 2000, y, …, in which every term after the first is 1 less than the product of its two immediate neighbors. For how many different values of x does the term 2001 appear somewhere in the sequence?

a) 1 b) 2 c) 3 d) 4 ~~e) more than 4~~

$a_{n+1} = a_n \cdot a_{n+2} - 1$ $\boxed{x,\ 2000,\ \frac{2001}{x},\ \frac{2001+x}{2000x},\ \frac{1+x}{2000},}\ x,\ 2000$

$\frac{a_{n+1}+1}{a_n} = a_{n+2}$ $\therefore y = \frac{2001}{x}$

$x = 2001,\ 1,\ \frac{2001}{2000\cdot 2001-1},$ and $2000\cdot2001-1$

A geometric sequence $a, ar, ar^2, \ldots$ is a sequence in which successive terms have a common ratio r. For example, the sequence $2, 10, 50, \ldots$ is a geometric sequence with common ratio $r = 5$ because $\frac{10}{2} = \frac{50}{10} = 5$.

$4y^2+8y+4 = 7y^2+y-7y-1$ $y = \frac{14 \pm \sqrt{196+60}}{6}$

$0 = 3y^2-14y-5$ 5, or $-\frac{1}{3}$

(c) If $y-1$, $2y+2$ and $7y+1$ are the first three terms of a geometric sequence, determine all possible values of y.

(d) For each of the values of y from (c), determine the 6th term of the geometric sequence $y-1,\ 2y+2,\ 7y+1, \ldots$.

(b) A geometric sequence has 20 terms.
The sum of its first two terms is 40.
The sum of its first three terms is 76.
The sum of its first four terms is 130.
Determine how many of the terms in the sequence are integers.

$\underbrace{16\ \ 24}_{40}\ \ 36\ \ 54$ $r = \frac{54}{36} = \frac{6}{4} = \frac{3}{2}$

$16\ \ 24\ \ 36\ \ 54\ \ 81\ \ \frac{243}{2}$

$\boxed{5}$

(A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

(b) Determine all possible values of r such that the three term geometric sequence $4, 4r, 4r^2$ is also an arithmetic sequence.
(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, 3, 5, 7, 9, 11 is an arithmetic sequence.)

$4r-4 = 4r^2-4r$

$4(r-1) = 4r(r-1)$

$4 = 4r$

$r = \boxed{1}$

Name: Claire Li Date: Dec 1

Section 2.3 Finite and Infinite Geometric Series

1. Find the sum for the following geometric series. Show all your work and steps:

a) $S=8+4+2+.....\frac{1}{128}$ $r=2$ $\frac{16-\frac{1}{128}}{2-1}=\boxed{15\frac{127}{128}}$	b) $S=3+6+12+.....3072$ $r=2$ $\frac{6144-3}{2-1}=\boxed{6141}$
c) $S=\sqrt{2}+2+2\sqrt{2}+.....256$ $r=\sqrt{2}$ $\frac{256\sqrt{2}-\sqrt{2}}{\sqrt{2}-1}=\frac{256\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{512+256\sqrt{2}}{1}$ $=\boxed{512+256\sqrt{2}}$	d) $S=-\frac{1}{8}+0.25+-0.5+.....256$ $r=-2$ $\frac{-512+\frac{1}{8}}{-2-1}=\frac{512-\frac{1}{8}}{3}=\frac{512}{3}-\frac{1}{24}$ $=\frac{4096}{24}$
e) $S=\frac{64}{81}+\frac{32}{27}+\frac{16}{9}+.....\frac{729}{16}$ $r=\frac{3}{2}$ $\frac{\frac{2187}{32}-\frac{64}{81}}{\frac{3}{2}-1}=\frac{2187}{16}-\frac{128}{81}=\frac{177147-2048}{1296}$ $=\frac{175099}{1296}$	f) $S=\frac{125}{64}-\frac{25}{16}+\frac{5}{4}-.....+\frac{1024}{3125}$ $r=\frac{4}{5}$ $\frac{\frac{4096}{15575}-\frac{125}{64}}{\frac{4}{5}-1}=\frac{\frac{125}{64}-\frac{4096}{15575}}{\frac{1}{5}}=\frac{625}{64}-\frac{4096}{3125}$ $=\frac{1690981}{200000}$

2. Given that each of the following series are infinite, determine the sum. Show all your work and steps.

$\frac{a}{1-r}$

a) $S=27+9+3+1+.......$ $r=\frac{1}{3}$ $\frac{27}{1-\frac{1}{3}}=\frac{27}{\frac{2}{3}}=27\cdot\frac{3}{2}=\boxed{\frac{81}{2}}$	b) $S=2+1.8+1.62+1.458+.......$ $\frac{1.8}{2}=\frac{18}{20}=\frac{9}{10}$ $\frac{2}{1-\frac{9}{10}}=\frac{2}{\frac{1}{10}}=\boxed{20}$
c) $S=0.4+\left(\frac{3}{100}+\frac{3}{1000}+\frac{3}{10000}+.......\right)$ $=0.4+\left(\frac{\frac{3}{100}}{1-\frac{1}{10}}\right)$ $=0.4+\frac{3}{100}\cdot\frac{10}{9}=\frac{12}{30}+\frac{1}{30}=\boxed{\frac{13}{30}}$	d) $S=11+16+8+4+2+.......$ Sum $=11+(16+8+4+2+\ldots$ $=11+\frac{16}{1-\frac{1}{2}}$ $=11+\frac{16}{\frac{1}{2}}$ $=11+32=\boxed{43}$
e) $S=256-128\sqrt{2}+128-64\sqrt{2}+........$ $r=-\frac{1}{\sqrt{2}}$ $=-\frac{\sqrt{2}}{2}$ $=\frac{256}{\frac{2+\sqrt{2}}{2}}$ $=256\cdot\frac{2(2-\sqrt{2})}{2+\sqrt{2}(2-\sqrt{2})}$ $=512-256\sqrt{2}$	f) $S=\frac{64}{125}+\frac{16}{25}+\frac{4}{5}+........$ $r=\frac{5}{4}$ $=\frac{\frac{64}{125}}{1-\frac{5}{4}}$ $=\frac{\frac{64}{125}}{-\frac{1}{4}}=\frac{64}{125}\cdot(-4)=\boxed{\frac{256}{125}}$

3. What is the sum of the following geometric series?

$12+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\ldots$ $r=\frac{3}{4}$.

$=12+\frac{\frac{3}{4}}{1-\frac{3}{4}}=$

$=12+3=\boxed{15}$ //

4. The sum of the first 8 terms of a geometric series is 1020 with a common ratio of -2. Determine the first term.

$\frac{a(r^n-1)}{r-1}=1020$

$\frac{a(256-1)}{-3}=1020$

$a(255)=-3060$

$a=\boxed{-\frac{104}{17}}$ //.

5. The sum of an infinite geometric series is 1 and the common ratio is $-\frac{2}{5}$, determine the 3rd term.

3rd term.

$\frac{a}{1-(-\frac{2}{5})}=\frac{a}{\frac{7}{5}}=1.$

$a\cdot\frac{5}{7}=1$ $a=\frac{7}{5}$

$\frac{7}{5}\cdot -\frac{14}{25}\cdot \boxed{\frac{28}{125}}$ //

6. Determine the 8th term of an infinite geometric series with $S_\infty=24$ and $r=\frac{3}{4}$

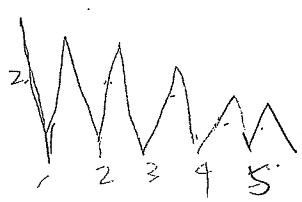
$\frac{a}{1-r}=24$

$\frac{a}{\frac{1}{4}}=24$

$a=6$

$6\cdot r^7=6\cdot\frac{2187}{16,384}=\boxed{\frac{6561}{8192}}$ //

7. A ball is dropped from a height of 2.0m to a floor. After each bounce, the ball rises to 63% of its previous height. What is the total vertical distance the ball has travelled after 5 bounces? What is the total vertical distance the ball travelled after it comes to rest?



$\left(\frac{2(0.63)(0.63^5-1)}{0.63-1}\right)\times 2-2).$

$=3.0674\times 4$

$=6.13$

$6.13+2=\boxed{8.13m}$ //.

$\left(\frac{a}{1-r}\right)\times2+2.$

$=\frac{1.26}{0.37}\times2+2$

$2)$ $\boxed{8.81}$ //

8. If the sum of a geometric series is given by the formula $S_n=4-8(-7)^{n-1}$, determine the value of t_5.

$\frac{a(r^n-1)}{r-1}=4-8(-7)^{n-1}$

$a(r^n-1)=4(r-1)-8(-7)^{n-1}(r-1).$

$\begin{cases} ar^n=-8(-7)^{n-1}(r-1). \\ -a=4(r-1.) \end{cases}$

$\begin{cases} 4(1-r)r^n=-8(-7)^{n-1}(\cancel{r-1}).\ ① \\ 4(1-r)=a \end{cases}$

$①=4r^n=8(-7)^{n-1}$

$r^n=2(-7)^{n-1}$

9. An oil well produces 30,000 barrels of oil during its first month of production. Suppose its production drops by 5% each month. Estimate the total production before the well runs dry.

$$\frac{30000}{1-0.95} = \frac{30000}{0.05} = \boxed{600000}$$

10. If the sum of "n" terms of a geometric series is $S_n = 2(3^n - 1)$, determine the 5th term of this series.

$$\frac{a(r^n-1)}{r-1} = 2(3^n-1)$$

$$\begin{cases} \frac{a}{r-1} = 2 \\ r = 3 \end{cases} \quad \frac{a}{3-1} = 2 \quad a = 4$$

5^{th} term $= 4 \cdot 3^4$

$= 4 \cdot 81$

$= \boxed{324}$

11. A contest winner is given two prizes to choose from. Prize A is given \$20,000,000 right away. Prize "B" is given \$1 in the first year, \$2 in the next year, \$4, \$8, ... each following year for the next 30 years. After how many years will sum of Prize B surpass the Prize A?

$$\frac{a(r^n-1)}{r-1} > 20,000,000$$

$$1(2^n-1) > 20000000$$

$$2^n > 20000001$$

$n = 21$

$\boxed{21 \text{ years.}}$

12. For any geometric series, what is the value of $S_{n+1} - S_n$ equal to?

$$\boxed{T_{n+1}}$$

13. The sum of the 1st and 2nd term of a geometric sequence is 4 and the sum of the 3rd and 4th term is 36. Determine the sum of the first 8 terms.

$$\underbrace{a \quad ar}_{4} \qquad \underbrace{ar^2 \quad ar^3}_{36} \qquad \begin{cases} a = 1 \\ r = 3 \end{cases}$$

$$\frac{a+ar}{a(r^2+r^3)} = \frac{4}{36}$$

$$\frac{1}{r^2} = \frac{1}{9} \quad r = \pm 3$$

$$\frac{3(r^n-1)}{2}$$

$$= \frac{3(3^8-1)}{2} = \boxed{9840}$$

14. Let $S_n = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....+\frac{1}{2^{n-1}}$. Prove algebraically that S_n is less than "2" for all values of "n". Justify your answer.

$$S_n = \frac{1\left(\frac{1}{2}^n - 1\right)}{\frac{1}{2}-1}$$

$$= \frac{1-\frac{1}{2^n}}{\frac{1}{2}}$$

$$= 2-\frac{1}{2^{n-1}}$$

$$\therefore \frac{1}{2^{n-1}} > 0$$

$$2-\frac{1}{2^{n-1}} < 2$$

15. Let $S_n = 1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+.....+\frac{1}{3^{n-1}}$. Prove algebraically that S_n is less than "1.5" for all values of "n". Justify your answer.

$$S_n = \frac{1\left(\frac{1}{3}^n - 1\right)}{\frac{1}{3}-1}$$

$$= \left(\frac{1}{3^n}-1\right)\left(-\frac{3}{2}\right)$$

$$= \frac{3}{2} - \frac{1}{3^{n-1}\cdot 2}$$

$$\frac{1}{3^{n-1}\cdot 2} > 0$$

$$\therefore \frac{3}{2} - \frac{1}{3^{n-1}\cdot 2} < \frac{3}{2}$$

16. For what values of "x" will the series have a finite sum? $1+\left(\frac{x-2}{3}\right)+\left(\frac{x-2}{3}\right)^2+\left(\frac{x-2}{3}\right)^3+....$

$$-1 < \frac{x-2}{3} < 1$$

$$-3 < x-2 < 3$$

① $-3 < x-2$, $-1 < x$

② $x-2 < 3$, $x < 5$

$$\boxed{-1 < x \text{ or } x < 5}$$

17. The geometric series $a+ar+ar^2+....$ has a sum of 7, and the terms involving odd powers of "r" have a sum of 3. What is $a+r$?

$$\frac{a}{1-r} = 7 \qquad \frac{ar}{1-r^2} = 3$$

$$a = 7-7r \qquad a = \frac{3-3r^2}{r}$$

$$7-7r = \frac{3-3r^2}{r}$$

$$7r-7r^2 = 3-3r^2$$

$$0 = 4r^2-7r+3$$

$$0 = (4r-3)(r-1)$$

$$r=1 \text{ or } r=\frac{3}{4}$$

$r \neq 1$, $r = \frac{3}{4}$

$$\frac{a}{1-\frac{3}{4}} = 7$$

$$4a = 7$$

$$a = \frac{7}{4}$$

$$\frac{7}{4}+\frac{3}{4} = \boxed{\frac{5}{2}}$$

18. In a sequence of numbers, the sum of the first "n" terms is equal to $5n^2+6n$. What is the sum of the 3rd, 4th, and 5th terms of the original sequence.

$$t_3+t_4+t_5 = S_5 - S_2$$

$$= 5\cdot 5^2 + 30 - 20 - 12$$

$$= 125 - 2$$

$$= \boxed{123}$$

Add this question to the test!!

$\frac{1+4(1+m)}{1}$

3-6. Evaluate $\left(\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\cdots\right)+\left(\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\cdots\right)+\cdots$. More formally, what is the sum of all fractions of the form $\frac{1}{(m+1)^{n+1}}$, where m and n range over the positive integers?	3-6.

Problem 3-6

Sums of non-negative numbers may be rearranged in any order whatsoever without affecting the value of the sum. Rewrite the sum as $S = \left(\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots\right)+\left(\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\ldots\right)+\left(\frac{1}{4^2}+\frac{1}{4^3}+\frac{1}{4^4}+\ldots\right)+\ldots$. Next we can use the formula for the sum of an infinite geometric series to write $S = \frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{(m)(m+1)}+\ldots = \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{m}-\frac{1}{m+1}\right)+\ldots$. This is a telescoping series in which most terms subtract out. The sum of the series' first m terms is $1-\frac{1}{m+1}$. Since m increases in size without bound, the value of S is $1-0=\boxed{1}$.

$$\left(\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+\cdots\right)+\cdots$$

$$=\left(\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots\right)+\left(\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots\right)+\cdots$$

$$=\frac{\frac{1}{2^2}}{1-\frac{1}{2}}+\frac{\frac{1}{3^2}}{1-\frac{1}{3}}+\frac{\frac{1}{4^2}}{1-\frac{1}{4}}+\frac{\frac{1}{5^2}}{1-\frac{1}{5}}+\cdots$$

$$=\frac{1}{2^2}\cdot\frac{2}{1}+\frac{1}{3^2}\cdot\frac{3}{2}+\frac{1}{4^2}\cdot\frac{4}{3}+\frac{1}{5^2}\cdot\frac{5}{4}$$

$$=\frac{1}{1\cdot2}+\frac{1}{3\cdot2}+\frac{1}{4\cdot3}+\frac{1}{5\cdot4}+\cdots$$

$$=\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\cdots$$

$$=1-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{5}}+\cdots$$

$$=1-\frac{1}{m+1}$$

$$=1.$$

$\frac{a}{1-r} = 7 \qquad \frac{ar}{1-r^2} = 3$

$a = 7-7r \qquad a = \frac{3-3r^2}{r}$

$\frac{3-3r^2}{r} = 7-7r$

$3-3r^2 = 7r-7r^2$

$4r^2-7r+3=0$

$4r \quad -3$

$r \quad -1$

$(4r-3)(r-1)=0 \qquad r \neq 1$

$r = 1 \text{ or } \frac{3}{4}$

$\frac{a}{1-\frac{3}{4}} = 7$

$a \cdot 4 = 7$

$a = \frac{7}{4}$

$\frac{7}{4} + \frac{3}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$

Amc 12 2007B #15

15. The geometric series $a + ar + ar^2 + \cdots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?

(A) $\frac{4}{3}$ (B) $\frac{12}{7}$ (C) $\frac{3}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

4. In a sequence of numbers, the *sum* of the first *n* terms is equal to $5n^2 + 6n$. What is the sum of the 3rd, 4th and 5th terms in the original sequence?

$t_3 + t_4 + t_5 = S_5 - S_2$

$= 5(5)^2 + 6(5) - (5(2)^2 + 12)$

$= 125 + 30 - 20 - 12$

$= \boxed{123}$

Name: Claire Date: Claire

Section 2.4 Recursive Sequences and Series

1. Given each recursive sequence, find the indicated term:

a) $t_1 = 3,\ \ t_{n+1} = 2\times t_n + 3,\ t_8 =$ 765 3 9 21 45 93 189 381 765	b) $t_1 = 2,\ t_2 = 3,\ \ t_{n+2} = t_n + 3t_{n+1},\ t_6 =$ 369 2 3 11 36 111 369
c) $t_1 = 3,\ \ t_{n+1} = 2\times t_n + 3,\ t_8 =$ 765 See (a)	d) $t_1 = 3,\ \ t_{n+1} = 2\times t_n + 3,\ t_8 =$ 765 see (a).
e) If $a_1 = 3$ and $a_{n+1} = 2a_n - 1$, what is a a_{10}? 3 5 9 17 33 65 129 257 513 $a_{10} = \boxed{1025}$ //	f) If $a_{10} = 10$ and $a_{n+1} = \frac{a_n - 1}{2} + 4$, what is a a_7? $10 = \frac{a_9 - 1}{2} + 4$ $13 = \frac{a_8 - 1}{2} + 4$ $19 = \frac{a_7 - 1}{2} + 4$ $12 = a_9 - 1$ $18 = a_8 - 1$ $15 = \frac{a_7 - 1}{2}$ $a_9 = 13$ $a_8 = 19$ $a_7 = \boxed{31}$ //
g) If $a_1 = 1,\ a_2 = 2$, and $a_n = 3a_{n-2} + a_{n-1}$ evaluate a_6 1, 2, 5, 11, 26, $\boxed{59}$, 137. // $a_6 = 59$	g) If $a_{10} = 10$ & $a_n = \frac{a_{n-1}}{2} + 4$, what is the value of a_7? $10 = \frac{a_9}{2} + 4$ $12 = \frac{a_8}{2} + 4$ $16 = \frac{a_7}{2} + 4$ $12 = a_9$ $16 = a_8$ $\boxed{24} = a_7$ //

2. Simplify the expression and find the sum: $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots\ldots + \frac{1}{19(20)}$

$= \frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \ldots + \cancel{\frac{1}{19}} - \frac{1}{20}$

$= \frac{1}{1} - \frac{1}{20}$

$= \frac{19}{20}$

3. The first term of a sequence is 2007. Each term, starting with the second, is the sum of the cubes of the digits of the previous term. What is the 2007th term?

2007 351 153 153 153 ---

$t_{2007} = \boxed{153}$ //

4. What is the sixth tem of a recursively defined sequece with its first term defined as $a_1 = 8$ and all subsequent terms defined as $a_n = (8 - a_{n-1})^2 + 7$?

8 7 8 7 8 7 ------

$t_6 = \boxed{7}$ //

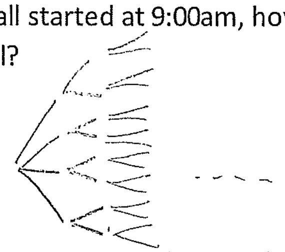
5. Each term in the serious below has the form $\frac{1}{(n)(n+3)}$. Find the sum of this series:

$$\frac{1}{(1)(4)} + \frac{1}{(4)(7)} + \frac{1}{(7)(10)} + \ldots \frac{1}{(298)(301)}$$

$= \frac{1}{3}\left(1 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} \cdots + \cancel{\frac{1}{298}} - \frac{1}{301}\right)$

$= \frac{1}{3}\left(1 - \frac{1}{301}\right) = \frac{300}{301} \cdot \frac{1}{3} = \boxed{\frac{100}{301}}$ //

6. A school has a phone tree that begins with the principle being responsible for calling four teachers. Each teacher afterwards is responsible for calling two other teachers. If each call takes exactly 2 minutes and the first call started at 9:00am, how long will it take to contact all the teachers if there are 250 teachers in the school?



$1 + 2^2 + 2^3 + 2^4 + 2^5 + \cdots$

$\frac{2^n - 2^2}{2-1} \geq 250 \qquad 2^n - 4 \geq 250 \qquad 2^n \geq 254$

$n = 8$

$8 \times 2 = \boxed{16 \text{ min}}$ //

1 4 8 16 32 64

7. A sequence of numbers $t_1, t_2, t_3 \ldots$ is defined by: $t_1 = 7$ and $t_{n+1} = \sqrt{|(t_n)^2 - 16|}$. What is the value of t_{80}?

7 $\sqrt{33}$ $\sqrt{17}$ 1 $\sqrt{15}$ 1 $\sqrt{15}$ 1 $\sqrt{15}$ ----

even term

$t_{80} = \boxed{1.}$ //

8. A function $f(x)$ has the following properties. Calculate $f(6)$

i) $f(1) = 1$, ii) $f(2x) = 4f(x) + 6$ iii) $f(x+2) = f(x) + 12x + 12$

$f(1+2) = f(1) + 12 \times 1 + 12$

$f(3) = 25$

$f(2 \times 3) = 4f(3) + 6$

$f(6) = 100 + 6$

$f(6) = \boxed{106}$ //

9. The sequence of numbers t_1, t_2, t_3, ... is defined by $t_1 = 2$ and $t_{n+1} = \dfrac{t_n - 1}{t_n + 1}$, for every positive integer "n".

Determine the numerical value of t_{999}

$[2 \quad \frac{1}{3} \quad -\frac{1}{2} \quad -3] \quad 2 \quad \frac{1}{3} \quad -\frac{1}{2} \quad -3 \ldots \quad 2 \quad \frac{1}{3} \quad \underset{t_{999}}{-\frac{1}{2}} \quad \underset{t_{1000}}{-3}$

$t_{999} = \boxed{-\frac{1}{2}}$

10. In seven term sequence, 5, p, q, 13, r, 40, x, each term after the third term is the sum of the preceding three terms. The value of x is

a) 21 b) 61 c) 67 d) 74 e) 80

$p+q+5=13$

$p+q=8$

$r = p+q+13$

$=21$

$x = 13+21+40 = 74$

11. The sequence 2, 5, 10, 50, 500,..... is formed so that each term after the second is the product of the two previous terms. The 15th term ends with exactly "k" zeroes. What is the value of "k"?

2 5 10 50 500 25000 12500000

1"0" 1"0" 2 3 5

1 1 2 3 5 8 13 21 34 55 89 144

233

$k = \boxed{233}$

12. The function $f(x)$ has the property that $f(x+y) = f(x) + f(y) + 2xy$, for all positive integers x and y. If $f(1) = 4$, then the numerical value of $f(8)$ is

a) 72 b) 84 c) 88 d) 64 e) 80

$f(1+1) = f(1) + f(1) + 2$

$f(2) = 10$

$f(2+2) = f(2) + f(2) + 8$

$f(4) = 28$

$f(4+4) = f(4) + f(4) + 32$

$f(8) = \boxed{88}$

13. A sequence t_1, t_2, ..., t_n, ... is defined as follows: i) $t_1 = 14$ ii) $t_k = 24 - 5t_{k-1}$ for each $k \geq 2$. For every postive integer n, t_n can be expressed as $t_n = p \cdot q^n r$, where p, q and r are constants. What is the value of $p+q+r$?

a) -5 b) -3 c) 3 d) 17 e) 31

14. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?

a) 160 b) 304 c) 28 d) 56 e) 64

$\underset{7th}{8} \quad \underset{8th}{4} \quad \underset{9th}{24} \quad \underset{10th}{56} \quad \underset{11th}{160}$

15. If $a_1 = \dfrac{1}{1-x}$, $a_2 = \dfrac{1}{1-a_1}$, and $a_n = \dfrac{1}{1-a_{n-1}}$, for $n \geq 2$, $x \neq 1$ and $x \neq 0$, then a_{107} is

a) $\dfrac{1}{1-x}$ b) x c) $-x$ d) $\dfrac{x-1}{x}$ e) $\dfrac{1}{x}$

$\dfrac{1}{1-x}$ $\dfrac{x-1}{x}$ x | $\dfrac{1}{1-x}$ $\dfrac{x-1}{x}$ x ---- $\dfrac{1}{1-x}$ $\boxed{\dfrac{x-1}{x}}$ x

a_1 a_2 a_3 | a_4 a_5 a_6 a_{106} $\boxed{a_{107}}$ a_{108}

16. The first term in a sequence of numbers is $t_1 = 5$. Succeesing terms are defined by the statement $t_n - t_{n-1} = 2n+3$ for $n \geq 2$. What is the value of t_{50}?

a) 2700 b) 2702 c) 2698 d) 2704 e)2706

5 $5+3+2\times2$ $12+3+3\times2$ ---
$= 12$ $=21$
t_1 t_2 t_3

$t_{50} = 5 + 2\times2 + 3\times2 + \cdots + 50\times2 + 3\times49$
$= 5 + \frac{(2+50)49}{2}\times2 + 3\times49$
$= 5 + 49(55) = 2700$

17. The Fibonacci sequence begins: 1, 1, 2, 3, 5, 8, 13, 21... (Each number beyond the second number is the sum of the previous two numbers.) The notation f_n means the n^{th} number, for example $f_4 = 3$ and $f_7 = 13$.

a) Which of the following terms in the Fibonacci sequence are odd? Explain your conclustions.

f_{38}, f_{51}, f_{150}, f_{200}, f_{300}

o o e o o e o o e ---

38 and 200 are not divisible by 3.

b) Which of the following terms in the Fibonacci sequence are divisible by 3? Explain your conclusions.

f_{48}, f_{75}, f_{196}, f_{379}, f_{1000}

11202210 | 11202210
8

48÷8 R0 ✓

75÷8 R3 X

196÷8 R0 ✓

379÷8 R3 X

1000÷8 R0 ✓

a_1 a_2 a_3 a_4 a_5 a_6 ... a_{100}

1 $99^{\frac{1}{3}}$ $99^{\frac{2}{3}}$ 99 $99^{\frac{4}{3}}$ $99^{\frac{5}{3}}$ ... $99^{\frac{99}{3}}$

13. Define a sequence of real numbers $a_1, a_2, a_3, \ldots$ by $a_1 = 1$ and $a_{n+1}^3 = 99a_n^3$ for all $n \geq 1$. Then a_{100} equals 99^{33}

(A) 33^{33} (B) 33^{99} (C) 99^{33} (D) 99^{99} (E) none of these

a_1 a_2 a_3 a_4 a_5 a_6

let $a_2 = x$ | 19 x $\frac{19+x}{2}$ $\frac{19+x}{2}$ $\frac{19+x}{2}$ $\frac{19+x}{2}$ ---- $a_9 = \frac{19+x}{2}$

20. The sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 19$, $a_9 = 99$, and, for all $n \geq 3$, a_n is the arithmetic mean of the first $n-1$ terms. Find a_2.

(A) 29 (B) 59 (C) 79 (D) 99 (E) 179

$a_9 = \frac{19+x}{2}$

$99 = \frac{19+x}{2}$

$198 = x+19$

$x = \boxed{179}$

3. (a) An infinite sequence $a_0, a_1, a_2, a_3, \ldots$ satisfies

$a_{1-0} + a_{1+0} = \frac{1}{2}a_2 + \frac{1}{2}a_0$

$2a_1 = \frac{1}{2}a_2$ $a_2 = 4$

$4a_1 = a_2$

$$a_{m-n} + a_{m+n} = \tfrac{1}{2}a_{2m} + \tfrac{1}{2}a_{2n}$$

for all non-negative integers m and n with $m \geq n \geq 0$.

Let $m=0$ and $n=0$

$a_{0-0} + a_{0+0} = \frac{1}{2}a_0 + \frac{1}{2}a_0$

$2a_0 = a_0$

$\boxed{a_0 = 0}$

$\therefore$ QED.

$a_2 + a_2 = \frac{1}{2}a_4 + \frac{1}{2}a_0$

$8 = \frac{1}{2}a_4$

$a_4 = 16$

$a_3 = \frac{1}{2}a_4 + 1$

$a_3 = 9$

(i) Show that $a_0 = 0$.

(ii) If $a_1 = 1$, determine the value of a_2 and the value of a_3. $\boxed{a_2 = 4,\ a_3 = 9}$

(b) An infinite sequence $b_0, b_1, b_2, b_3, \ldots$ satisfies

$$b_{m-n} + b_{m+n} = b_{2m} + b_{2n}$$

for all non-negative integers m and n with $m \geq n \geq 0$. Prove that all terms in the sequence have the same value.

Let $m=n$

$b_{m-m} + b_{m+m} = b_{2m} + b_{2m}$

$b_0 + b_{2m} = 2b_{2m}$

$b_0 = b_{2m}$

$\therefore b_0 = b_2 = b_4 = b_6$ ----

$b_{2m} = 2b_m - b_0 \leftarrow b_{2m} = b_0$

$2b_0 = 2b_m$

$b_0 = b_m$

$\therefore$ All terms are equal to b_0

$\therefore$ QED.

$t_1=150$ $t_4=2B-150$ $t_7=750-8B$

$t_2=B$ $t_5=300-3B$ $t_8=13B-1200$

COMC 2004 (Challenge) $t_3=150-B$ $t_6=5B-450$

4. In a *sumac sequence*, t_1, t_2, t_3, …, t_m, each term is an integer greater than or equal to 0. Also, each term, starting with the third, is the difference of the preceding two terms (that is, $t_{n+2} = t_n - t_{n+1}$ for $n \geq 1$). The sequence terminates at t_m if $t_{m-1} - t_m < 0$. For example, 120, 71, 49, 22, 27 is a sumac sequence of length 5. $B=\left\lfloor \frac{150}{\phi} \right\rfloor$ where $\phi=\frac{1+\sqrt{5}}{2}$ $B=92$

   (a) Find the positive integer B so that the sumac sequence 150, B, … has the maximum possible number of terms.

   $B=93$ 150 93 57 36 21 15 6 9 −3 ← 9.

   $B=92$ 150 92 58 34 24 10 14 −4 ← 8.

   $B=93$

   (b) Let m be a positive integer with $m \geq 5$. Determine the number of sumac sequences of length m with $t_m \leq 2000$ and with no term divisible by 5.

4. A function $f(x)$ has the following properties:

   i) $f(1) = 1$

   ii) $f(2x) = 4f(x) + 6$

   iii) $f(x+2) = f(x) + 12x + 12$

   Calculate $f(6)$.

$f(2)=4f(1)+6$
$=10$
$f(4)=4f(2)+6$
$=46$

$f(6)=f(4)+12\times4+12$
$=46+48+12$
$=\boxed{106}$

6. The sequence of numbers $t_1, t_2, t_3, \ldots$ is defined by $t_1 = 2$ and $t_{n+1} = \frac{t_n - 1}{t_n + 1}$, for every positive integer n.

   Determine the numerical value of t_{999}. $2, \frac{1}{3}, -\frac{1}{2}, -3, 2, \frac{1}{3}, -\frac{1}{2}, -3$ ……

   $t_{999} = \boxed{-\frac{1}{2}}$

7 If $a_{10} = 10$ and $a_n = \frac{a_{n-1}}{2} + 4$, what is the value of a_7?

A) $\frac{257}{32}$ B) $\frac{33}{4}$ C) 10 D) 24 E) 64

$a_{10}=\frac{a_9}{2}+4$
$10=\frac{a_9}{2}+4$
$a_9=12$

$a_9=\frac{a_8}{2}+4$
$a_8=16$

$a_8=\frac{a_7}{2}+4$
$12=\frac{a_7}{2}$
$a_7=24$

Amc 12 2006 #25 challenging

25. A sequence $a_1, a_2, \ldots$ of non-negative integers is defined by the rule $a_{n+2} = |a_{n+1} - a_n|$ for $n \geq 1$. If $a_1 = 999$, $a_2 < 999$, and $a_{2006} = 1$, how many different values of a_2 are possible?

$a_3=|a_2-a_1|$

(A) 165 **(B)** 324 **(C)** 495 **(D)** 499 **(E)** 660

a_1 a_2 a_3 a_4

999 x $999-x$ $|999-2x|$

$999-x-999+2x$

x

$999-2$ $1998-3x$

$999-x+999-2x$

Amc 12 2008 #17

$\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\cdots+\frac{1}{19}-\frac{1}{20}$

Simplify: $\frac{1}{1(2)}+\frac{1}{2(3)}+\frac{1}{3(4)}+\cdots+\frac{1}{19(20)}$. $\boxed{\frac{19}{20}}$

A) 1 B) $\frac{9}{10}$ C) $\frac{19}{20}$ D) $\frac{11}{30}$ E) Answer Not Given

17. Let $a_1, a_2, \ldots$ be a sequence of integers determined by the rule $a_n = a_{n-1}/2$ if a_{n-1} is even and $a_n = 3a_{n-1}+1$ if a_{n-1} is odd. For how many positive integers $a_1 \le 2008$ is it true that a_1 is less than each of a_2, a_3, and a_4? $\therefore 3a_1+1 = 2(2k+1) = 4k+2$

(A) 250 (B) 251 (C) 501 (D) 502 (E) 1004

$3a_1 = 4k+1$

$a_1 = 4k+3$

$4k+3 \le 2008$

$\boxed{k \le 501.25}$

① $a_1 < a_2$
if a_1 is even, $a_1 < \frac{a_1}{2}$ ✗
if a_1 is odd, $a_2 = 3a_1+1$ ✓

② $a_1 < a_3$
$a_3 = \frac{a_2}{2} = \frac{3a_1+1}{2}$
$a_1 < \frac{3a_1+1}{2}$ ✓

③ $a_1 < a_4$
if a_3 is even, $a_4 = \frac{a_3}{2} = \frac{3a_1+1}{4}$
$a_1 < \frac{3a_1+1}{4}$ impossible
a_3 is odd $a_3 = \frac{3a_1+1}{2}$

4. In a *sumac sequence*, $t_1, t_2, t_3, \ldots, t_m$, each term is an integer greater than or equal to 0. Also, each term, starting with the third, is the difference of the preceding two terms (that is, $t_{n+2} = t_n - t_{n+1}$ for $n \ge 1$). The sequence terminates at t_m if $t_{m-1} - t_m < 0$. For example, 120, 71, 49, 22, 27 is a sumac sequence of length 5.

(a) Find the positive integer B so that the sumac sequence 150, B, ... has the maximum possible number of terms.

(b) Let m be a positive integer with $m \ge 5$. Determine the number of sumac sequences of length m with $t_m \le 2000$ and with no term divisible by 5.

see page 6. Q4

6. The sequence of numbers $t_1, t_2, t_3, \ldots$ is defined by $t_1 = 2$ and $t_{n+1} = \frac{t_n - 1}{t_n + 1}$, for every positive integer n.

Determine the numerical value of t_{999}. see page 6. Q6

7. The sequence of numbers $\ldots, a_{-3}, a_{-2}, a_{-1}, a_0, a_1, a_2, a_3, \ldots$ is defined by $a_n - (n+1)a_{2-n} = (n+3)^2$, for all integers n. Calculate a_0. $n=2: a_2 - 3a_0 = 25$ $a_2 = -26$

$n=0: a_0 - a_2 = 9$ $a_2 - 3a_2 - 27 = 25$ $a_0 = -26+9$

$a_0 = a_2 + 9$ $-2a_2 = 52$ $= -17$

4. (a) Consider the sequence $t_1 = 1$, $t_2 = -1$ and $t_n = \left(\frac{n-3}{n-1}\right)t_{n-2}$ where $n \ge 3$. What is the value of t_{1998}?

$1 \quad -1 \quad 0 \quad -\frac{4}{3} \quad 0 \quad -\frac{4}{5} \quad 0 \quad -\frac{4}{7} \quad 0 \quad -\frac{4}{9}$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9 \quad t_{10}$

$t_{1998} = \boxed{-\frac{4}{1997}}$

(b) The nth term of an arithmetic sequence is given by $t_n = 555 - 7n$.
If $S_n = t_1 + t_2 + \ldots + t_n$, determine the smallest value of n for which $S_n < 0$.

$= 555 - 7(1) + 555 - 7(2) + 555 - 7(3) + \cdots 555 - 7(n) < 0$

$555n - 7(1+2+\cdots+n) < 0$

$555n - \frac{7(n+1)n}{2} < 0$

$555n < \frac{7(n+1)n}{2}$

$1110 < 7n+7$

$1103 < 7n$

$\frac{1103}{7} < n$

$n_{min} = \boxed{158}$

10. A *Skolem sequence* of order n is a sequence $(s_1, s_2, \ldots, s_{2n})$ of $2n$ integers satisfying the conditions:

i) for every k in $\{1, 2, 3, \ldots, n\}$, there exist exactly two elements s_i and s_j with $s_i = s_j = k$, and

ii) if $s_i = s_j = k$ with $i < j$, then $j - i = k$.

For example, $(4, 2, 3, 2, 4, 3, 1, 1)$ is a Skolem sequence of order 4.

(a) List all Skolem sequences of order 4.

a, (1 1 4 2 3 2 4 3)
(4 2 3 2 4 3 1 1)
(2 3 2 4 3 1 1 4)
(3 4 1 1 3 2 4 2)

(b) Determine, with justification, all Skolem sequences of order 9 which satisfy all of the following three conditions:

5 9 1 1 7 4 6 9 5 8 7 6 4 2 3 2 3 8
5 9 1 1 9 6 2 5 7 8 2 9 6 3 4 7 3 4

None satisfies all three

I) $s_3 = 1$, ✓

II) $s_{18} = 8$, and ✓

III) between any two equal even integers, there is exactly one odd integer. X

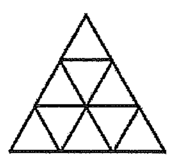
(c) Prove that there is no Skolem sequence of order n, if n is of the form $4k+2$ or $4k+3$, where k is a non-negative integer.

$n=4k+2$ $\quad$ $n=4k+3$

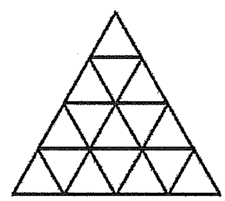
$n(3n+1)=(4k+2)(3(4k+2)+1)$ $\quad$ $n(3n+1)=(4k+3)(3(4k+2)+1)$

10. Suppose that n is a positive integer. Consider an upward-pointing equilateral triangle of side length n, cut up into unit triangles, as shown.

$n = 3$

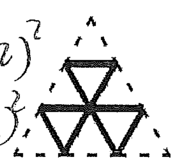


$n = 4$



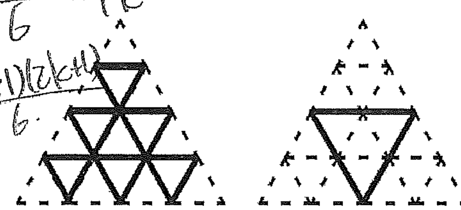
For each n, let $f(n)$ represent the total number of downward-pointing equilateral triangles of *all* sizes. For example, $f(3) = 3$ and $f(4) = 6 + 1 = 7$, as illustrated below.

$n = 3$



$f(3) = 3$

$n = 4$



$f(4) = 6 + 1 = 7$

$f(2k) = f(2k-1) + k^2$

$= \frac{k(k-1)(2k-1)}{6} + k^2$

$= \frac{k(k+1)(2k+1)}{6}$

$f(2k-1)$
$= f(2k-2) + (k-1)^2$
$= f(2k-3) + (k-1)^2 + (k-2)^2$
$= f(1) + 1^2 + 2^2 + \cdots + (k-1)^2$
$= \boxed{\frac{k(k-1)(2k-1)}{6}}$

(a) Determine the values of $f(5)$ and $f(6)$. $\quad f(5)=23 \quad f(6)=48$

odd n
let $n=2k-1$

(b) Prove that $f(2k) = f(2k-1) + k^2$ for each positive integer $k \geq 1$.

$f(n)=\frac{n(n^2-1)}{8}$

(c) Determine, with justification, all positive integers n for which $f(n)$ is divisible by n.

n are be any positive integer

even n
let $n=2k$. $\quad f(n)=\frac{n(n+2)(n+1)}{24}$
$k=\frac{n}{2}$

Name: Claire Li Date: Dec. 11

Math 10 Enriched: Section 2.5 Sigma Notations and Summation

1. Indicate the number of terms in each series. If the series is geometric, find the common ratio.

a) $\sum_{x=4}^{11} x+3$ # of terms: $11-4+1=\boxed{8}$	b) $\sum_{x=-2}^{13} 3(2)^{x-5}$ $13-(-2)+1=\boxed{16}$ $R=2$	c) $\sum_{x=-1}^{99} 5^x+3$ $99-(-1)+1$ $=\boxed{101}$ terms. $R=5$	d) $\sum_{x=a}^{9} 2x=78$ $a=3$ or -4 $2(x+x+1\ldots 9)=78$ $\frac{(a+9)(9-a+1)}{2}=39$. # of terms: $-a^2+a+90=78$ $a^2-a-12=0$ $\boxed{7 \text{ or } 14}$
e) $\sum_{x=3}^{a} x^2=814$ $3^2+4^2+\cdots+a^2=814$ $1^2+2^2+\cdots+a^2=819$ $a=13$. # of terms $13-3+1$ $=\boxed{11}$	f) $\sum_{x=n-2}^{n+6} x+7$ $n+6-(n-2)+1$ $=\boxed{9}$ terms.	g) $\sum_{x=2n-1}^{n+1} 3^x-3x$ $n+1-(2n-1)+1$ $=\boxed{-n+3}$ terms	h) $\sum_{x=-4}^{a} 3(-2)^x=8192.063$ $a-(-4)+1=\boxed{a+5}$ terms.

2. Write the series corresponding to each expression and then find the sum:

a) $\sum_{x=5}^{10} x^2$ $=5^2+6^2+7^2+8^2+9^2+10^2$ $=\frac{10(21)(11)}{6}-1^2-2^2-3^2-4^2$ $=385-30=\boxed{355}$	b) $\sum_{x=2}^{8} x-4$ $=\sum_{x=2}^{8} x-4\times 7$ $=2+3+4+5+6+7+8-28$ $=\boxed{7}$
c) $\sum_{x=-2}^{4} 3^x$ $=\frac{1}{3^2}+\frac{1}{3}+1+3+3^2+3^3+3^4$ $=\frac{3^5-\frac{1}{9}}{3-1}$ $=\frac{242\frac{8}{9}}{2}=\boxed{121\frac{4}{9}}$	d) $\sum_{x=1}^{6} 5(2)^{x-1}$ $=5\times(2^0+2^1+2^2+\cdots+2^5)$ $=5\times\left(\frac{2^6-1}{2-1}\right)$ $=5\times 63=\boxed{315}$
e) $\sum_{x=-3}^{3} x^x-1$ $=(-3)^{-3}+(-2)^{-2}+(-1)^{-1}+0^0+1^1+2^2+3^3-1\times 7$ $=-\frac{1}{27}+\frac{1}{4}\cancel{-1}\cancel{+1}+1+4+27-7$ $=25+\frac{27}{108}-\frac{4}{108}=\boxed{25\frac{23}{108}}$	f) $\sum_{x=1}^{1000}\left(\frac{9}{10^x}\right)$ $=\frac{9}{10}+\frac{9}{10^2}+\frac{9}{10^3}+\cdots\frac{9}{10^{1000}}$ $=\frac{\frac{9}{10^{1001}}-\frac{9}{10}}{\frac{1}{10}-1}$ $=\left(\frac{9}{10^{1001}}-\frac{9}{10}\right)\times\left(-\frac{10}{9}\right)=\boxed{1-\frac{1}{10^{1000}}}$

3. Express each series using sigma notations in simplest form:

2×3^{12}

a) $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$ $=\sum_{x=-1}^{5}\frac{1}{2^x}$	b) $2-6+18-54+162-486+.....+1062882$ $\sum_{x=0}^{12} 2\times(-3)^x$
c) $a+a+d+a+2d+a+3d+...a+(n-1)d$ $\sum_{x=0}^{n-1} a+xd$	d) $a+ar+ar^2+ar^3+ar^4+...+ar^{n-1}$ $\sum_{x=0}^{n-1} ar^x$
e) $\sqrt{2}+2+2\sqrt{2}+4+4\sqrt{2}+....+128\sqrt{2}$ $\sum_{x=1}^{15}\sqrt{2}^x$	f) $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}$ $\sum_{x=-1}^{5}\frac{1}{2^x}$
g) $-\sqrt{5}-\sqrt{10}-2\sqrt{5}-2\sqrt{10}-.....$ $\sum_{x=0}^{\infty}-\sqrt{5}\times\sqrt{2}^x$	h) $2+\frac{2}{1.01}+\frac{2}{1.01^2}+\frac{2}{1.01^3}+.....$ $\sum_{x=0}^{\infty}\frac{2}{1.01^x}$

4. Evaluate each of the following series:

a) $\sum_{n=1}^{2001} a_n.$ $= a_1 + a_2 + a_3 + \cdots a_{2001}$	b) $\sum_{i=1}^{\infty} \frac{1}{5^i}$ $= \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots$ $= \frac{\frac{1}{5}}{1-\frac{1}{5}}$ $= \boxed{\frac{1}{4}}$ //
c) $\sum_{k=1}^{3} \frac{1}{2k}$ $= \frac{1}{2} + \frac{1}{4} + \frac{1}{6}$ $= \frac{6}{12} + \frac{3}{12} + \frac{2}{12}$ $= \boxed{\frac{11}{12}}$ //	d) $\sum_{i=1}^{10} \frac{10}{i}$ $= 10 + \frac{10}{2} + \frac{10}{3} + \frac{10}{4} + \frac{10}{5} + \frac{10}{6} + \frac{10}{7} + \frac{10}{8} + \frac{10}{9} + \frac{10}{10}$ $= 10 + 5 + \frac{10}{3} + \frac{5}{2} + 2 + \frac{5}{3} + \frac{10}{7} + \frac{5}{4} + \frac{10}{9} + 1$ $= 18 + \frac{2425}{252}$ $= \boxed{27.62}$ //

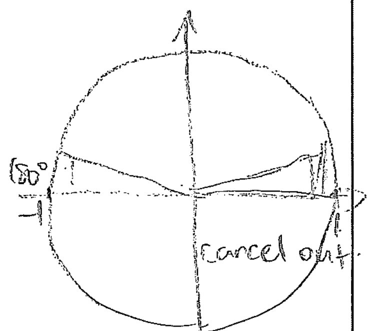
5. Solve for "x"

a) $\sum_{z=1}^{x} 5(2)^z = 1270$ $\frac{1270}{5} = 2 + 2^2 + 2^3 + \cdots 2^x$ $254 = \frac{2^{x+1} - 2}{2-1}$ $256 = 2^{x+1}$ $x+1 = 8$ $x = 7.$ $x = \boxed{7}$ //	b) $\sum_{z=1}^{3} x^{z-1} = 7$ $7 = x^0 + x^1 + x^2$ $7 = 2^0 + 2^1 + 2^2$ $7 = 1 + 2 + 4$ $x = \boxed{2}$ //
c) $\sum_{n=1}^{\infty} 15x\left(x^2\right)^{n-1} = 4$ $\frac{4}{15} = x(x^2)^0 + x(x^2)^1 + x(x^2)^2$ $\frac{4}{15x} = x^0 + x^2 + x^4 + x^6 \cdots$ $\frac{4}{15x} = \frac{x^0}{1-x^2}$ $15x = 4 - 4x^2$ $4x^2 + 15x - 4 = 0$ $(4x-1)(x+4) = 0$ $x = -4 \text{ or } \frac{1}{4}$	d) $\sum_{z=1}^{\infty} 4(x)^{z-1} = 3$ $x^0 + x^1 + x^2 + x^3 = \frac{3}{4}$ $\frac{x^0}{1-x} = \frac{3}{4}$ $4 = 3 - 3x$ $3x = -1$ $x = \boxed{-\frac{1}{3}}$ //

e) $\sum_{\theta=0}^{180^\circ} \cos\theta = x$

$x = \cos 0 + \cos 1 + \cos \cdots \cos 180$

$= 0$



f) $\sum_{n=1}^{\infty} 3(x)^{n-1} + \sum_{n=1}^{\infty} 5(x^2)^{n-1} = 18$

$3(x^0+x^1+\cdots)+5(x^0+x^2+\cdots)=18$

$3\left(\frac{x^0}{1-x}\right)+5\left(\frac{x^0}{1-x^2}\right)=18$

$\frac{3}{1-x}+\frac{5}{1-x^2}=18$

$\frac{3-3x^2}{1-x}+5 = 18-18x^2$

$3+3x = 13-18x^2$

$18^2+3x-10=0.$

$(6x+5)(3x-2)=0$

$x = \boxed{\frac{2}{3} \text{ or } -\frac{5}{6}}$

6. Find the sum of the series $\sum_{m=1}^{n}(-1)^m$ if "m" is odd? OR if "m" is even.

odd: $(-1)^1+(-1)^3+\cdots+(-1)^n$

$= n\times(-1)$

$= \boxed{-n}$

even: $(-1)^2+(-1)^4+\cdots(-1)^n$

$= n\cdot\times 1$

$= \boxed{n}$

7. Find the sum of the following series: $\sum_{x=1}^{10}\left(2^x-5\right)$

$= \sum_{x=1}^{10} 2^x - 5\times 10$

$= (2^1+2^2+\cdots 2^{10})-50$

$= \frac{2^{10}\cdot 2-2}{2-1}-50$

$= 1022-50 = \boxed{972}$

8. Which expression represents the sum of the series given by: $\sum_{x=3}^{12} 6(3)^{x-1}$

a) $27(3^9-1)$ b) $27(3^{10}-1)$ c) $108(3^9-1)$ d) $108(3^{10}-1)$ e) $729(3^{11}-1)$

$6\times(3^2+3^3+3^4+\cdots 3^{11})$

$= \overset{3}{\cancel{6}}\times\left(\frac{3^{12}-3^2}{\cancel{3-1}}\right)$

$= 3\times 3^2(3^{10}-1)$

$= 27(3^{10}-1)$

CNML 2005/06 #6

36 | Evaluate: $\sum_{a=1}^{5}\sum_{b=0}^{3}(a+b)^2$ =

$$(1+0)^2+(1+1)^2+(1+2)^2+(1+3)^2 = 1^2+2^2+3^2+4^2$$
$$(2+0)^2+(2+1)^2+(2+2)^2+(2+3)^2 = 2^2+3^2+4^2+5^2$$
$$\ldots\ldots$$
$$(5+0)^2+(5+1)^2+(5+2)^2+(5+3)^2 = 5^2+6^2+7^2+8^2$$
$$= 1^2+(2^2)\times 2+(3^2)\times 3+(4^2)\times 4+4(5)^2+6^2\times 3+7^2+2+8^2$$
$$= 1+8+27+64+100+108+98+64 = \boxed{470}$$

3-6. If $f(n) = \dfrac{\log_{10} n}{\log_{10}(2006n - n^2)}$, find the sum of all 2005 terms of the series

$(2006n-n^2)^{fn} = n$

$$f(1) + f(2) + f(3) + \ldots + f(2004) + f(2005).$$

2007 MOSP Practice test #11

11.2. Suppose that a sequence $a_1, a_2, a_3, \ldots$ satisfies

$$0 < a_n \leq a_{2n} + a_{2n+1} \qquad (*)$$

for all $n \geq 1$. Determine if the series $\sum_{n=1}^{\infty} a_n$ converges or not. What if $a_1, a_2, a_3, \ldots$ is a sequence of positive numbers satisfies

$$0 < a_n \leq a_{n+1} + a_{n^2} \qquad (**)$$

instead?

2007 MOST TEST 2002 (p17/21)

2. Let p be a prime number greater than 5. For any integer x, define

$$f_p(x) = \sum_{k=1}^{p-1} \frac{1}{(px+k)^2}.$$

Prove that for all positive integers x and y, the numerator of $f_p(x) - f_p(y)$, when written in lowest terms, is divisible by p^3.

4. Consider the polynomials

$$f(x) = \sum_{k=1}^{n} a_k x^k \quad \text{and} \quad g(x) = \sum_{k=1}^{n} \frac{a_k}{2^k - 1} x^k,$$

where $a_1, a_2, \ldots, a_n$ are real numbers and n is a positive integer. Show that if 1 and 2^{n+1} are zeros of g then f has a positive zero less than 2^n.

Claire

Sequence and Series Problems

Euclid 2024

4.

7 3 10 13 23 36

2. a. In a sequence with six terms, each term after the second is the sum of the previous two terms. If the fourth term is 13 and the sixth term is 36, what is the first term?

$\boxed{7}$

b. For some real number $r \neq 0$, the sequence $5r$, $5r^2$, $5r^3$ has the property that the second term plus the third term equals the square of the first term. What is the value of r?

$5r^2+5r^3=25r^2$

$5r^3=20r^2$

$5r=20$

$r=\boxed{4}$

Euclid 2023

c. An arithmetic sequence with 7 terms has first term d^2 and common difference d. The sum of the 7 terms in the sequence is 756. Determine all possible values of d.

$d^2 \quad d^2+d \quad d^2+2d \cdots d^2+$

$7d^2+21d=756$

$d^2+3d=108$

$d^2+3d-108=0$

d -12, d 9

21, 4, 9 12

$(d-12)(d+9)=0$

$d=\boxed{12 \text{ or } -9}$

Euclid 2002:

5. a. A list a_1, a_2, a_3, a_4 of rational numbers is defined so that if one term is equal to r, then the next term is equal to $1+\dfrac{1}{1+r}$. For example, if $a_3=\dfrac{41}{29}$, then $a_4=1+\dfrac{1}{1+(41/29)}=\dfrac{99}{70}$. If $a_3=\dfrac{41}{29}$, what is the value of a_1

$\frac{41}{29}=1+\frac{1}{1+r}$

$\frac{12}{29}=\frac{1}{1+r}$

$12+12r=29$

$12r=17$

$r=\frac{17}{12}$

$a_2=\frac{17}{12}$

$\frac{17}{12}=1+\frac{1}{1+r}$

$\frac{5}{12}=\frac{1}{1+r}$

$5+5r=12$

$5r=7$

$r=\frac{7}{5}$

$a_1=\boxed{\frac{7}{5}}$

Euclid 2021

6. a. Suppose that $n>5$ and that the numbers $t_1, t_2, t_3, \ldots, t_{n-2}, t_{n-1}, t_n$ form an arithmetic sequence with n terms. If $t_3=5$, $t_{n-2}=95$, and the sum of all n terms is 1000, what is the value of n?

$t_1=5-2d \cdot t_n=95+2d \quad \frac{100n}{2}=1000 \quad n=\boxed{20}$

b. Suppose that a and r are real numbers. A geometric sequence with first term a and common ratio r has 4 terms. The sum of this geometric sequence is $6+6\sqrt{2}$. A second geometric sequence has the same first term a and the same common ratio r, but has 8 terms. The sum of this second geometric sequence is $30+30\sqrt{2}$. Determine all possible values for a.

$\frac{a-ar^4}{1-r}=6+6\sqrt{2}$

$\frac{a-ar^8}{1-r}=30+30\sqrt{2}$

$\frac{-ar^8+ar^4}{1-r}=24+24\sqrt{2}$

$\frac{ar^4(1-r^4)}{(1-r)}=24+24\sqrt{2}$

$r^4(6+6\sqrt{2})=24+24\sqrt{2}$

$r^4=4$

$r=\pm\sqrt{2}$

$a=\boxed{12 \text{ or } -6-4\sqrt{2}}$

AMC 12 2009

Let $a+ar_1+ar_1^2+ar_1^3+\cdots$ and $a+ar_2+ar_2^2+ar_2^3+\cdots$ be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is r_1, and the sum of the second series is r_2. What is r_1+r_2?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1+\sqrt{5}}{2}$ (E) 2

$\frac{a}{1-r_1}=r_1$

$a=r_1-r_1^2$

$\frac{a}{1-r_2}=r_2$

$a=r_2-r_2^2$

$r_1-r_1^2=r_2-r_2^2$

$r_1-r_2=r_1^2-r_2^2$

$\cancel{(r_1-r_2)}=\cancel{(r_1-r_2)}(r_1+r_2)$

$1=r_1+r_2$

2024: Problem 12

The first three terms of a geometric sequence are the integers a, 720, and b, where $a<720<b$. What is the sum of the digits of the least possible value of b?

(A) 9 (B) 12 (C) 16 (D) 18 (E) 21

$720=2^4 \cdot 3^2 \cdot 5$

360 720 1440

2023 Problem 12

What is the value of

$2(9\times10)^2 = 2\times90^2 = 16200$

$9^2(2\times81-1) = 81(162) = 13041$

$$2^3 - 1^3 + 4^3 - 3^3 + 6^3 - 5^3 + \cdots + 18^3 - 17^3?$$

(A) 2023 **(B)** 2679 **(C)** 2941 **(D)** 3159 **(E)** 3235

2023 Problem 20:

Rows 1, 2, 3, 4, and 5 of a triangular array of integers are shown below:

row 7th: 1 6+5 15+14 20+19 15+14 6+5 1

```
        1                1
      1   1              2  >2x+1
    1   3   1            5  >2x+2
  1   5   5   1          12 >2x+3
1   7   11  7   1        27 >2x+4
 1   9   19  19   9   1  58
```

$a_1 = 1$
$a_2 = 2$
$a_3 = 2\times2+1=5$
$a_4 = 5\times2+2$
$a_n = 2\times a_{n-1} + n-2$

Each row after the first row is formed by placing a 1 at each end of the row, and each interior entry is 1 greater than the sum of the two numbers diagonally above it in the previous row. What is the units digit of the sum of the 2023 numbers in the 2023rd row?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

$2^{2023} - 2023$

2024 Problem 21:

Suppose that $a_1 = 2$ and the sequence (a_n) satisfies the recurrence relation

$a_2 = x$ $\quad \frac{x-1}{1} = \frac{1+1}{2}$ $\quad x = 2$ $\quad a_2 = 2$ $\quad a_3 = 3$ $\quad a_4 = 4$

$$\frac{a_n - 1}{n-1} = \frac{a_{n-1} + 1}{(n-1) + 1}$$

for all $n \geq 2$. What is the greatest integer less than or equal to

$\sum_{n=1}^{100} a_1^2 + a_2^2 + \cdots a_{100}^2 = 1^2 + 2^2 + \cdots 100^2 = \frac{100(201)(101)}{6} = 338350$

$$\sum_{n=1}^{100} a_n^2?$$

(A) 338,550 **(B)** 338,551 **(C)** 338,552 **(D)** 338,553 **(E)** 338,554

$\sin(a+b) = \sin a\cos b + \sin b\cos a$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a\cos b + \sin b\cos a}{\cos a\cos b - \sin a\sin b}$

Amc 12 2009 Q25

pattern of 24 terms

25 The first two terms of a sequence are $a_1 = 1$ and $a_2 = \frac{1}{\sqrt{3}}$. For $n \geq 1$,

$2009 \equiv 17 \pmod{24}$ $\quad$ $\tan 180 = 0$

$$a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}.$$

$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

What is $|a_{2009}|$?

(A) 0 **(B)** $2-\sqrt{3}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** 1 **(E)** $2+\sqrt{3}$

$\tan a = 1$ $\quad a = 45°$ $\quad \tan b = \frac{1}{\sqrt{3}}$ $\quad b = 30°$

tan 45, tan 30, tan 75, tan 105, tan 180, tan 105, tan 105, tan 30, tan 135, tan 165, tan 120, tan 105, tan 45, tan 130, tan 155, tan 110